

A new general relativistic planetary orbitography platform

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IAU General Assembly PhD prize talk Fri 5 August

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|--------------------------|--|------|------|
| <input type="checkbox"/> | On the energy integral for first post-Newtonian approximation
J O'Leary, JM Hill, JC Bennett
Celestial Mechanics and Dynamical Astronomy 130 (7), 1-9 | 6 | 2018 |
| <input type="checkbox"/> | Dynamical properties of the Molniya satellite constellation: long-term evolution of the semi-major axis
J Daquin, EM Alessi, J O'Leary, A Lemaitre, A Buzzoni
Nonlinear dynamics 105 (1) | 5 | 2021 |
| <input type="checkbox"/> | An Australian Conjunction Assessment Service
JCS Bennett, M Lachut, D Kooymans, A Pollard, C Smith, S Flegel, ...
Proceedings of the 2019 AMOS Conference, Maui, HI, USA, 17-20 | 3 | 2019 |
| <input type="checkbox"/> | Progress in a new conjunction and threat warning service for space situational awareness
JC Bennett, M Lachut, D Kucharski, SK Flegel, M Möckel, J Allworth, ...
Proceedings of the 19th Advanced Maui Optical Space Surveillance ... | 3 | 2018 |
| <input type="checkbox"/> | Post-Newtonian satellite orbits
J O'Leary, JM Hill
Astrophysics and Space Science 363 (10), 1-6 | 2 | 2018 |
| <input type="checkbox"/> | Generalized transformations and coordinates for static spherically symmetric general relativity
JM Hill, J O'Leary
Royal Society open science 5 (4), 171109 | 2 | 2018 |
| <input type="checkbox"/> | Numerical conservation of exact and approximate first post-Newtonian energy integrals
J O'Leary, JM Hill | 1 | 2018 |
| <input type="checkbox"/> | An application of symplectic integration for general relativistic planetary orbitography subject to non-gravitational forces
J O'Leary, JP Barriot
Celestial Mechanics and Dynamical Astronomy 133 (11), 1-22 | 2021 | |
| <input type="checkbox"/> | General Relativistic and Post-Newtonian Dynamics for Near-Earth Objects and Solar System Bodies
J O'Leary
Springer Nature | 2021 | |

THE IAU 2000 RESOLUTIONS FOR ASTROMETRY, CELESTIAL MECHANICS, AND METROLOGY IN THE RELATIVISTIC FRAMEWORK: EXPLANATORY SUPPLEMENT

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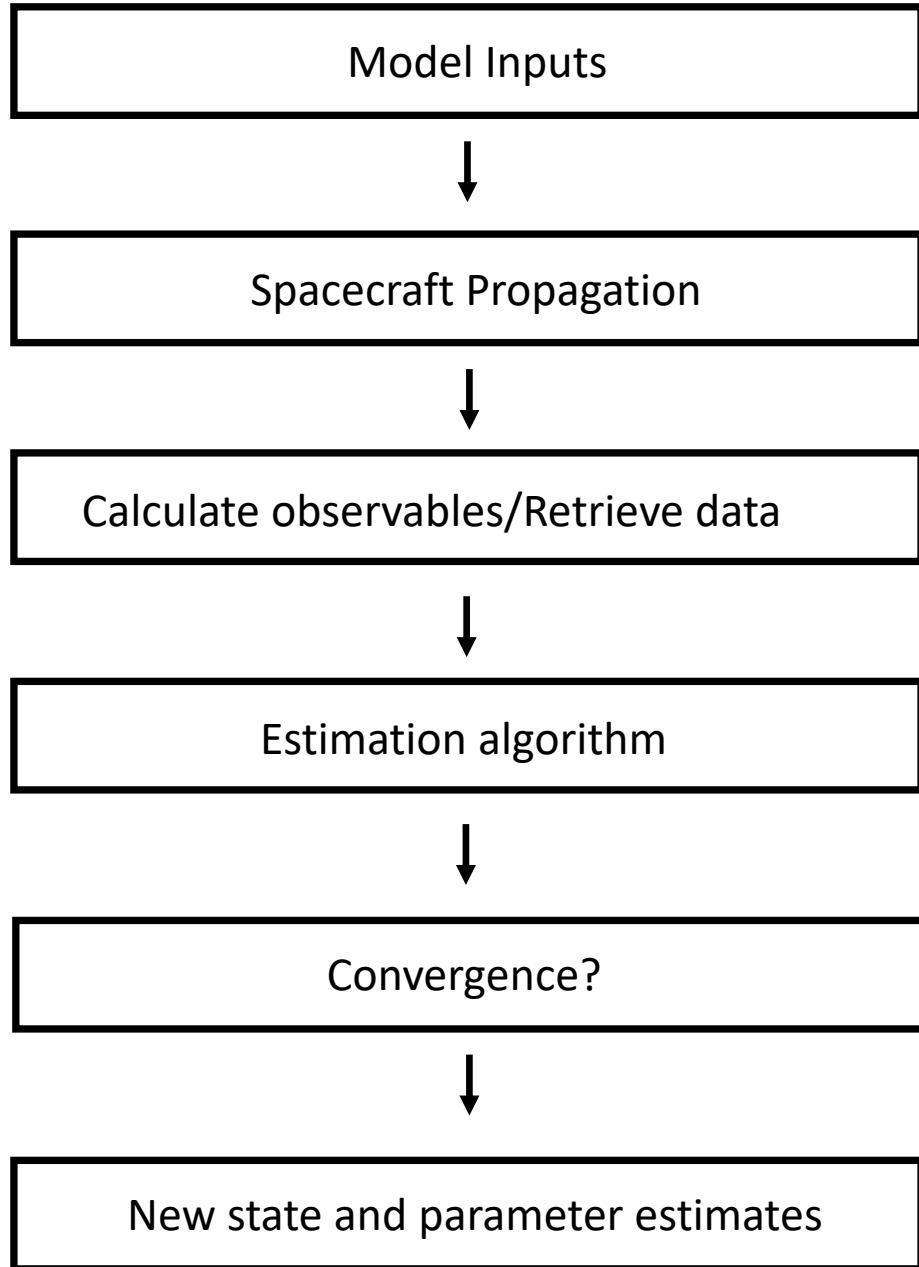
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1. INTRODUCTION

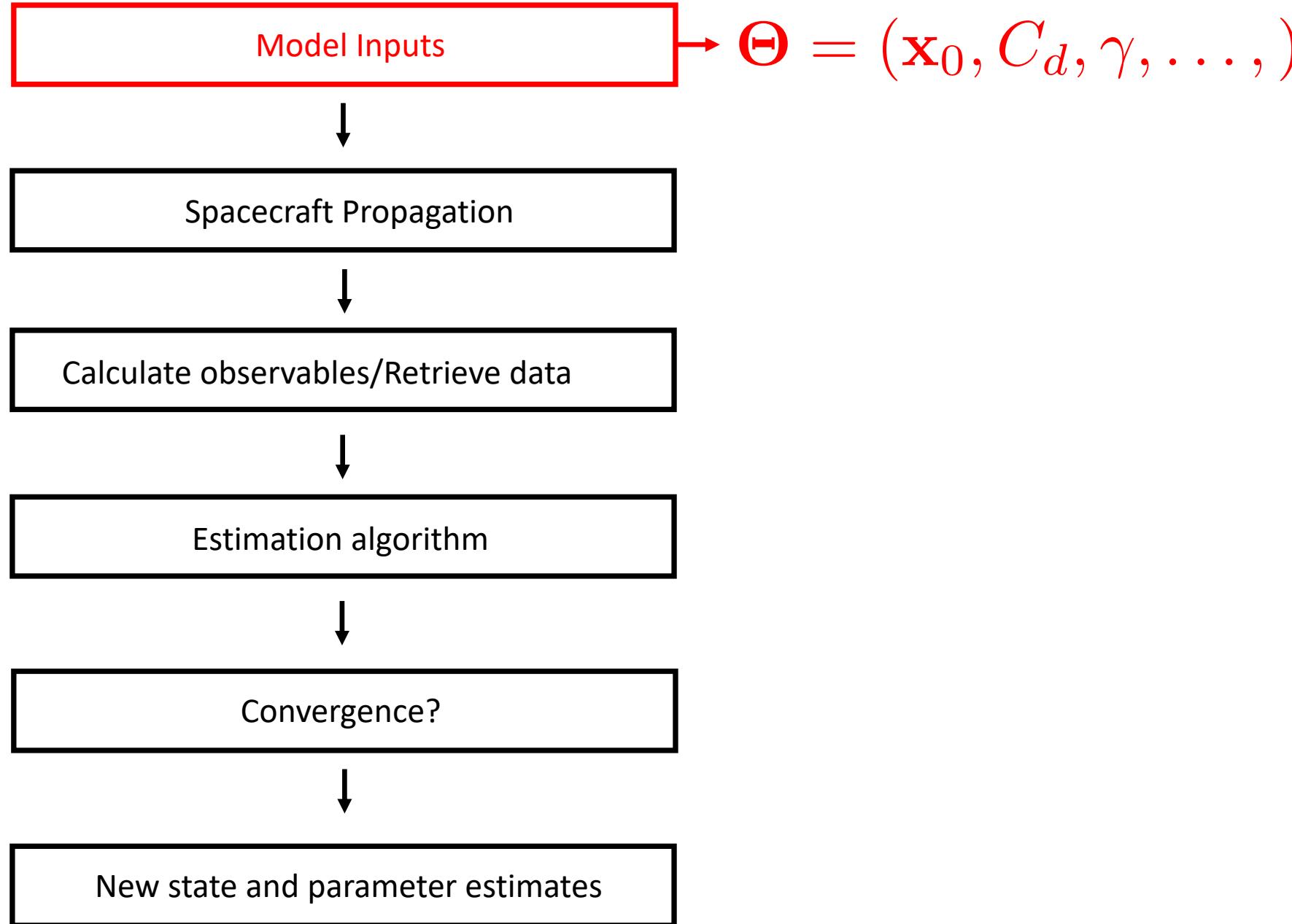
It is clear that, beyond some threshold of accuracy, any astronomical problem has to be formulated within the framework of Einstein's theory of gravity (general relativity theory, or GRT). Many high-precision astronomical techniques have already passed this threshold. For example,

The IAU resolutions on relativity represent a post-Newtonian framework allowing one to model any kind of astronomical observation in a rigorous, self-consistent manner with accuracies that are sufficient for the coming decades. They replace the old IAU relativistic framework, which was insufficient for many reasons discussed above. These new resolutions, however, are not expected to lead to dramatic changes. In fact, in many fields of application the models presently in use are already fully compatible with the new IAU resolutions, and in this sense the IAU resolutions officially fix the status quo. Let us give some examples of this.

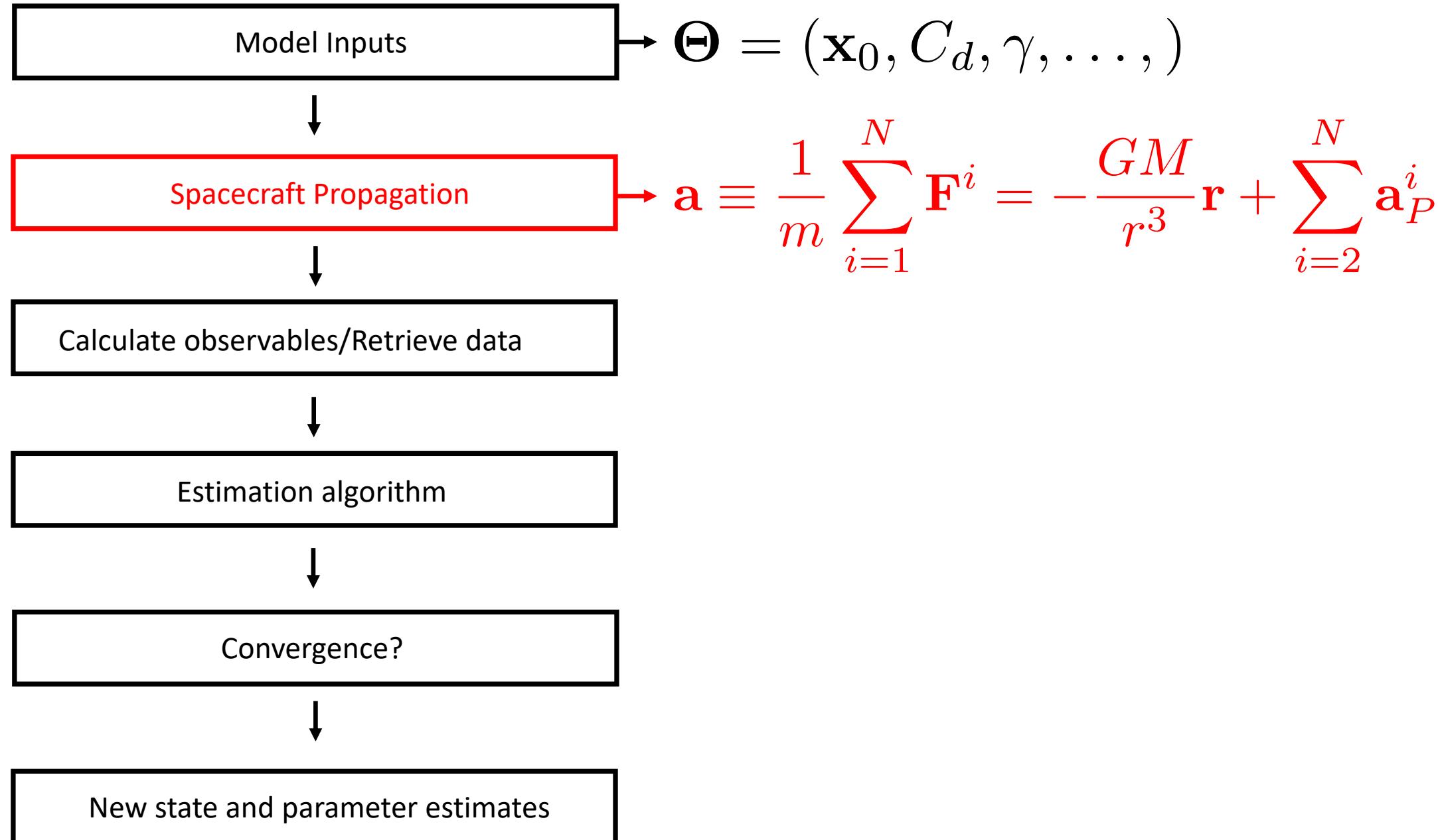
Orbit Determination Tool:



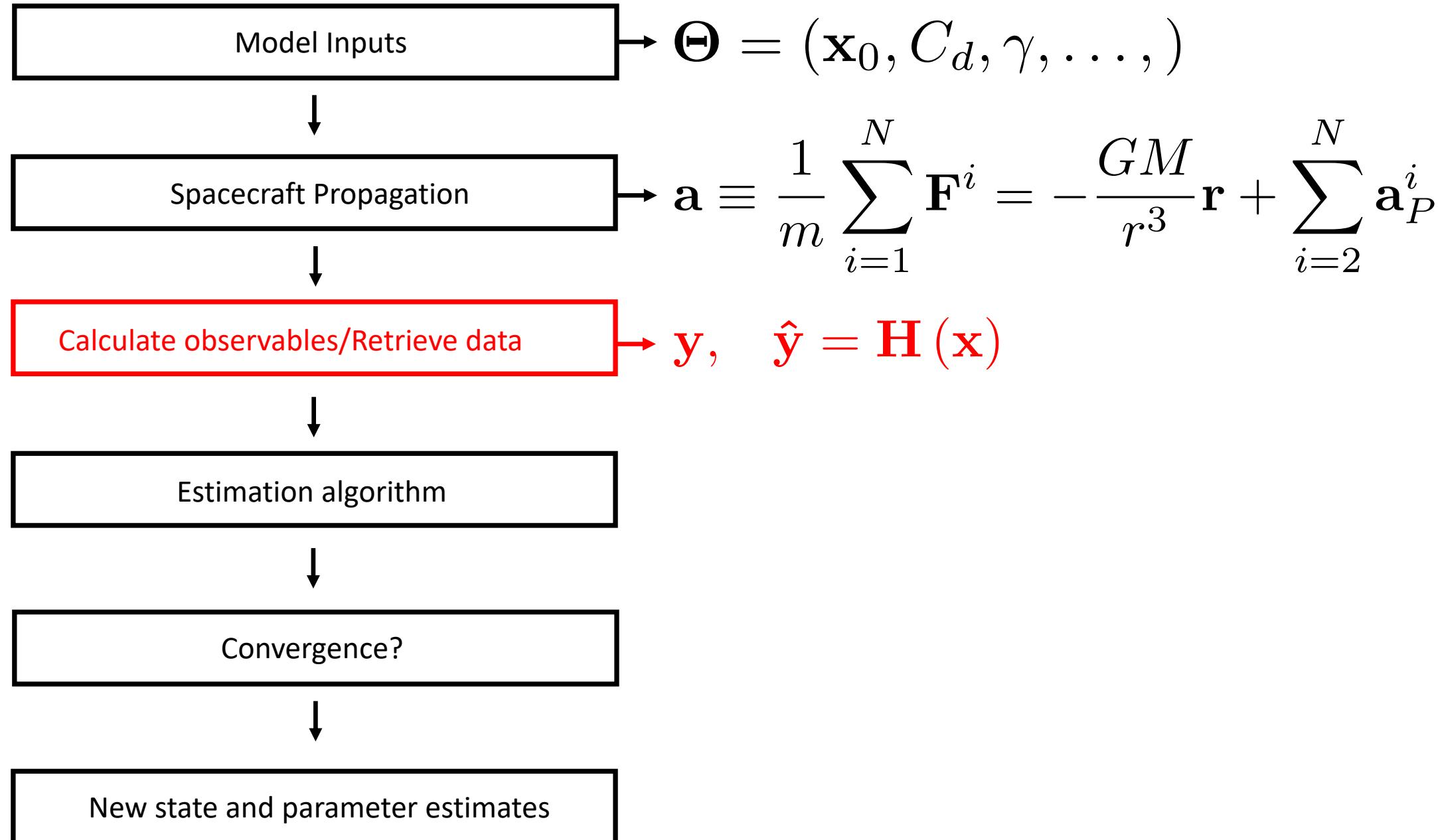
Orbit Determination Tool:



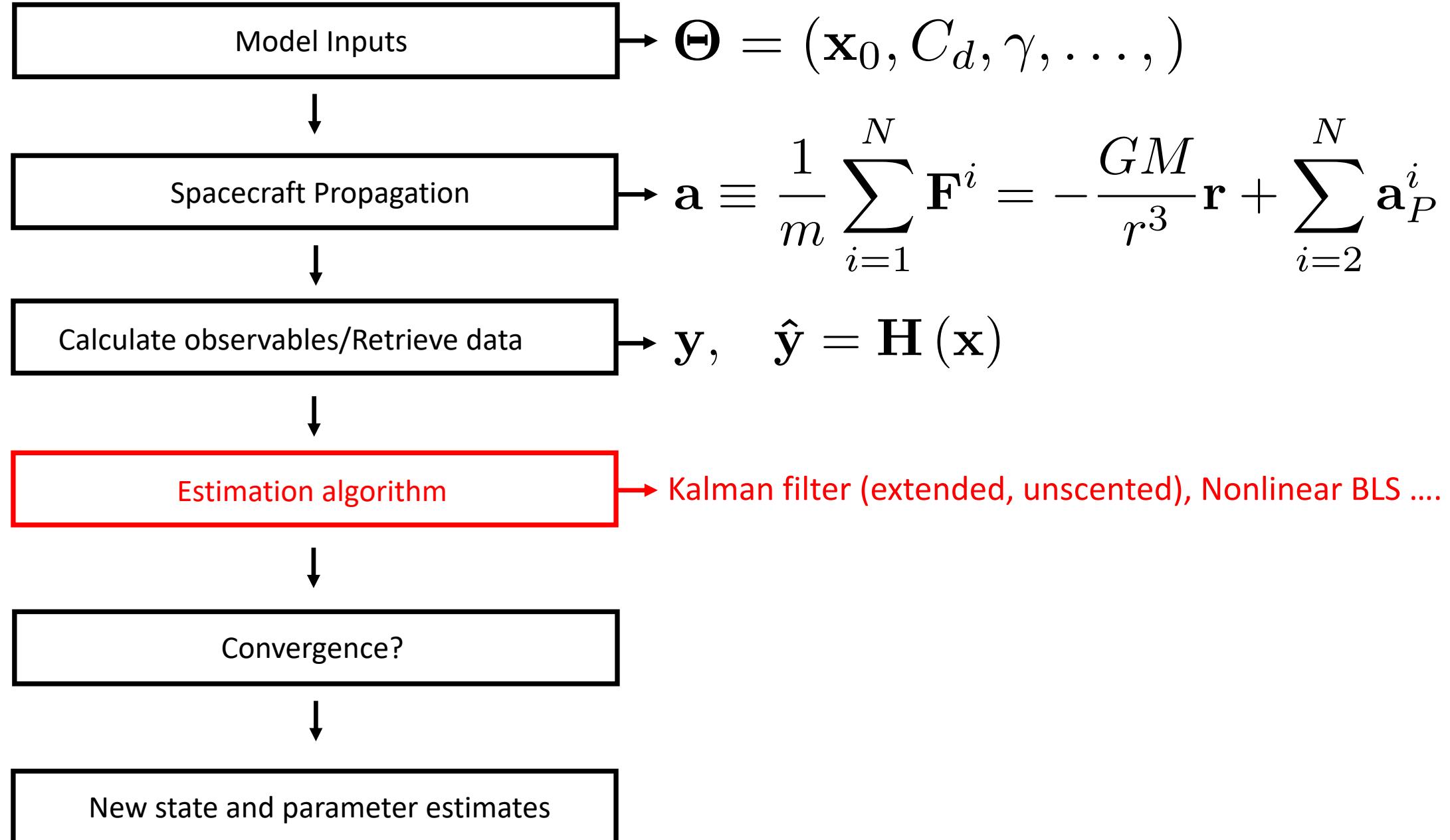
Orbit Determination Tool:



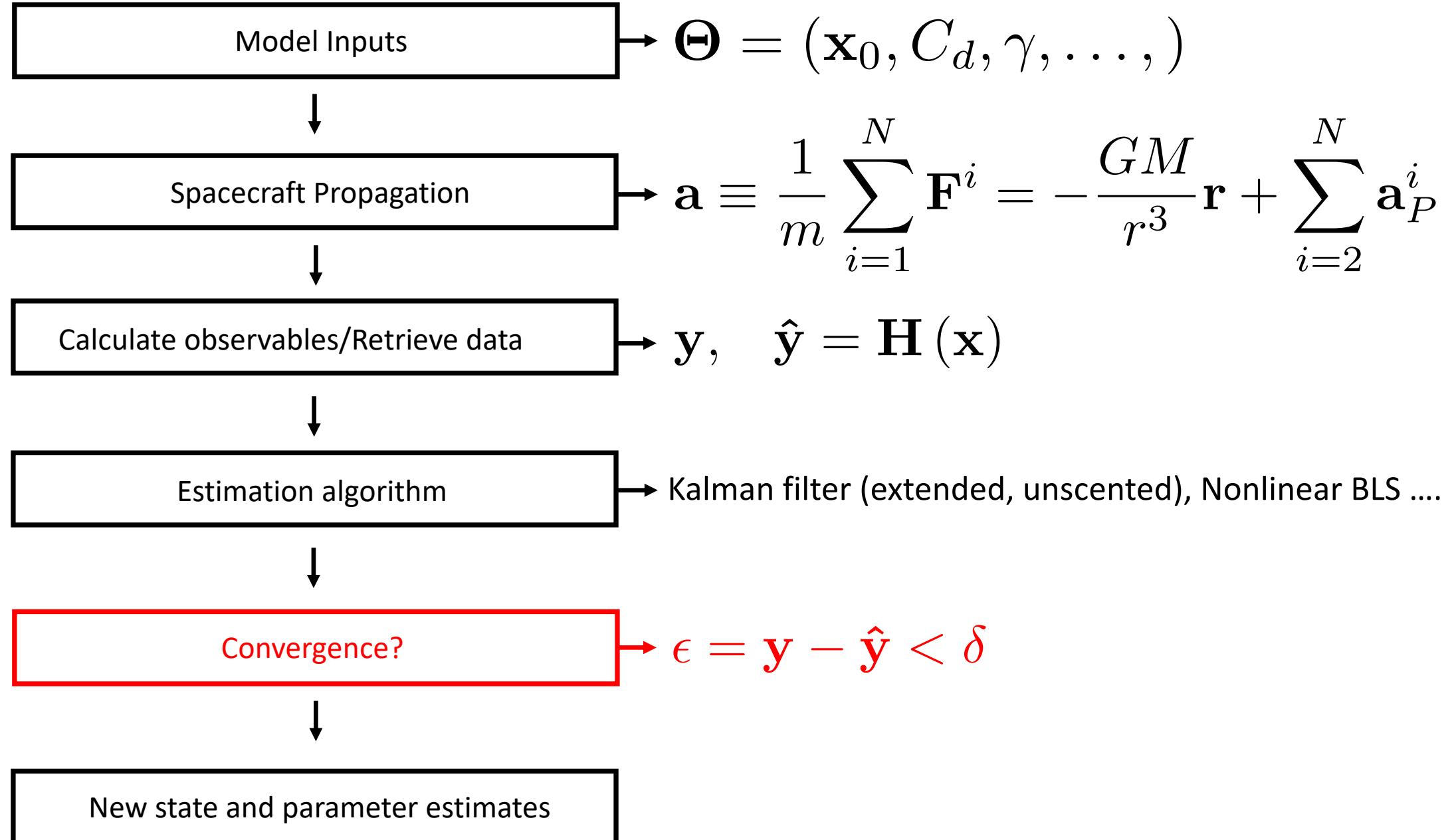
Orbit Determination Tool:



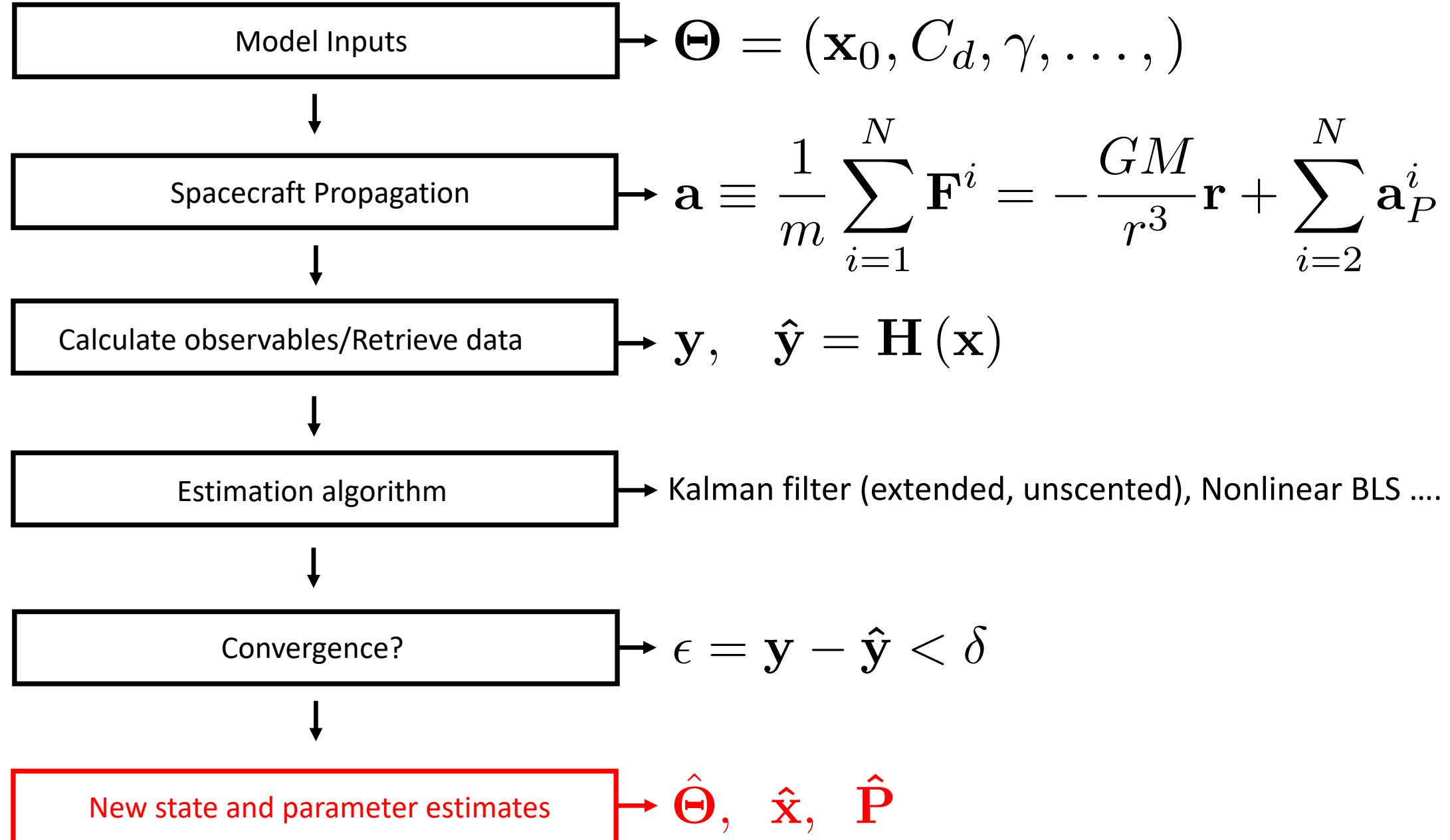
Orbit Determination Tool:



Orbit Determination Tool:



Orbit Determination Tool:



General Relativistic Accelerometer-based Propagation Environment (GRAPE):

Model Input

$$\rightarrow g_{\mu\nu}$$

Spacecraft Propagation

$$\rightarrow \frac{d^2x^\alpha}{d\tau^2} = -\Gamma_{\mu\nu}^\alpha \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} + f^\alpha$$

$$f^\alpha \equiv \mathcal{K}_\beta (g^{\alpha\beta} - u^\alpha u^\beta / c^2)$$

$$\rightarrow \frac{1}{c^2} \frac{d^2x^i}{dt_e^2} = -\Gamma_{00}^i - 2\Gamma_{0k}^i \beta^k - \Gamma_{jk}^i \beta^j \beta^k + \Gamma_{00}^0 \beta^i + 2\Gamma_{0k}^0 \beta^i \beta^k$$

$$+ \Gamma_{jk}^0 \beta^i \beta^j \beta^k - \frac{1}{c^2} (f^0 \beta^i - f^i) \left(\frac{d\tau}{dt_e} \right)^2$$

$$\rightarrow u_\alpha f^\alpha = 0 \implies \mathcal{I} \equiv u_\alpha u^\alpha = c^2$$

Determining the components of \mathcal{K}^α

In the local frame (L) of the spacecraft we have

$$u^\alpha = (c, 0, 0, 0)$$

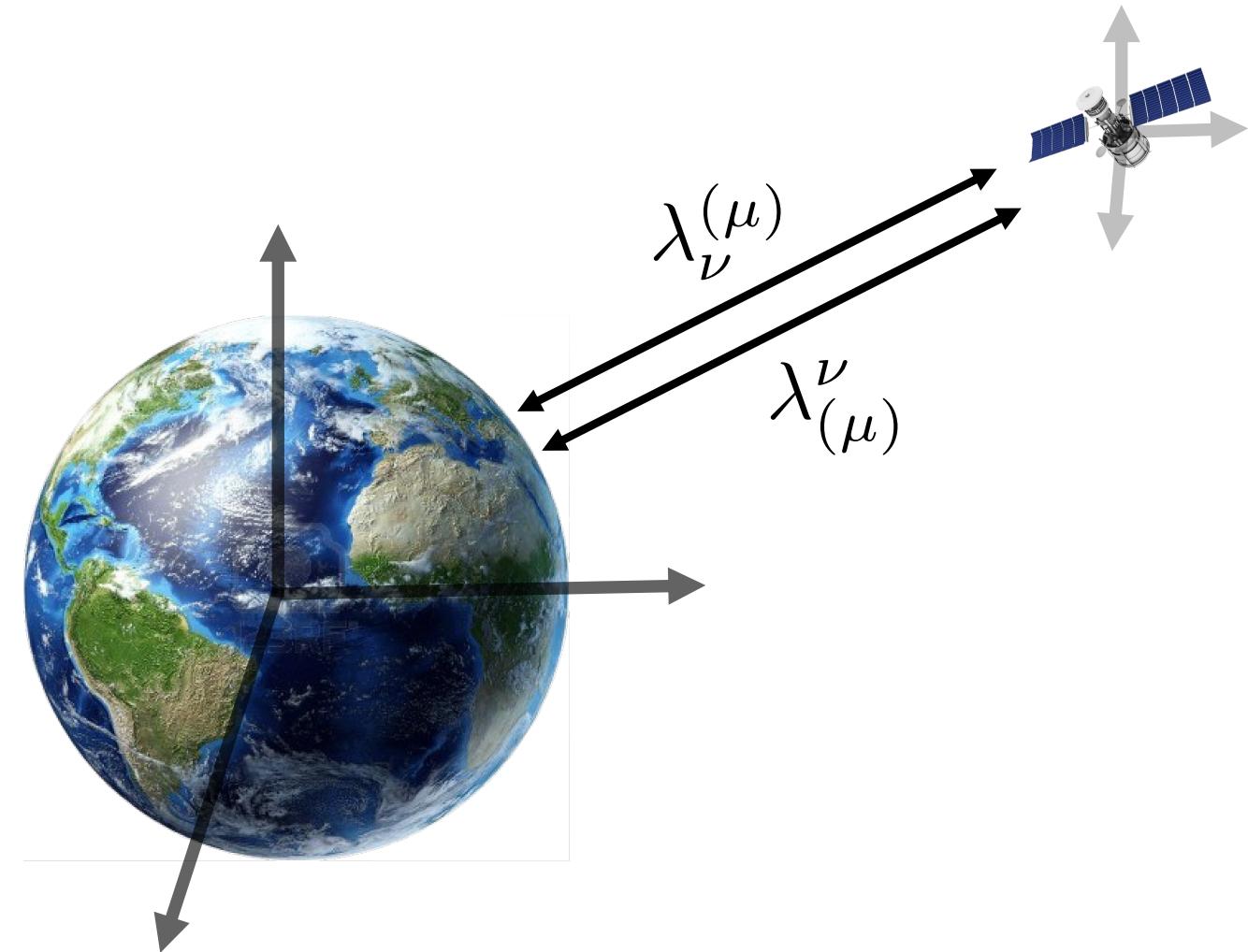
so that given the definition

$$u_\alpha f^\alpha = 0$$

we find

$$f_L^\alpha = (0, \mathcal{K}_L^i)$$

where the \mathcal{K}_L^i are provided to GRAPE by end-users using either accelerometer data or specific dynamical models. From there, the “global” force are obtained using the tetrad formalism (See O’Leary & Barriot, 2021 for further explicit details).



$$g_{\mu\nu} \lambda_{(\alpha)}^\mu \lambda_{(\beta)}^\nu = \eta_{\alpha\beta}$$

$$dx^{(\mu)} = \lambda_\nu^{(\mu)} dx^\nu$$

GRAPE: Earth, Mercury and Solar orbiter examples

$$g_{00} = \left(\frac{1 - \rho_s/\rho}{1 + \rho_s/\rho} \right)^2, \quad g_{ij} = \delta_{ij} (1 + \rho_s/\rho)^4,$$

$$(ds)^2 = \left(\frac{1 - \rho_s/\rho}{1 + \rho_s/\rho} \right)^2 c^2 dt^2 - (1 + \rho_s/\rho)^4 \delta_{ij} dx^i dx^j,$$

$$\begin{aligned} \frac{d^2 x^0}{d\tau^2} &= -4 \left(\frac{\rho_s}{\rho^3} \right) \left(\frac{x_i u^i}{w_1} \right) u^0 + f^0, \\ \frac{d^2 x^i}{d\tau^2} &= -2\rho_s \left\{ \rho^3 (\rho - \rho_s) w_2 (u_0 u^0) x^i - (\rho^3 w_3)^{-1} \right. \\ &\quad \times \left[(u_i u^i - u_j u^j) x^i + 2 (x_j u^j) u^i \right] \right\} + f^i, \end{aligned}$$

Table 1 Molniya (M), Parker Solar Probe (PSP) and Mercury Planetary Orbiter (MPO) orbital inclination, i , period T , semi-major axis a and geocentric, heliocentric and hermiocentric apoapsis r_a and periapsis r_p radial distances, respectively

Spacecraft	i [deg]	r_a [km]	r_p [km]	T [sec]	a [km]
Molniya	63.4	43370	7650	39480.49	25060
Parker Solar Probe	3.4	110×10^6	6.7×10^6	7687503.77	5835×10^4
Mercury Planetary Orbiter	90	3940	2920	8503.42	3430

GRAPE Test Examples:

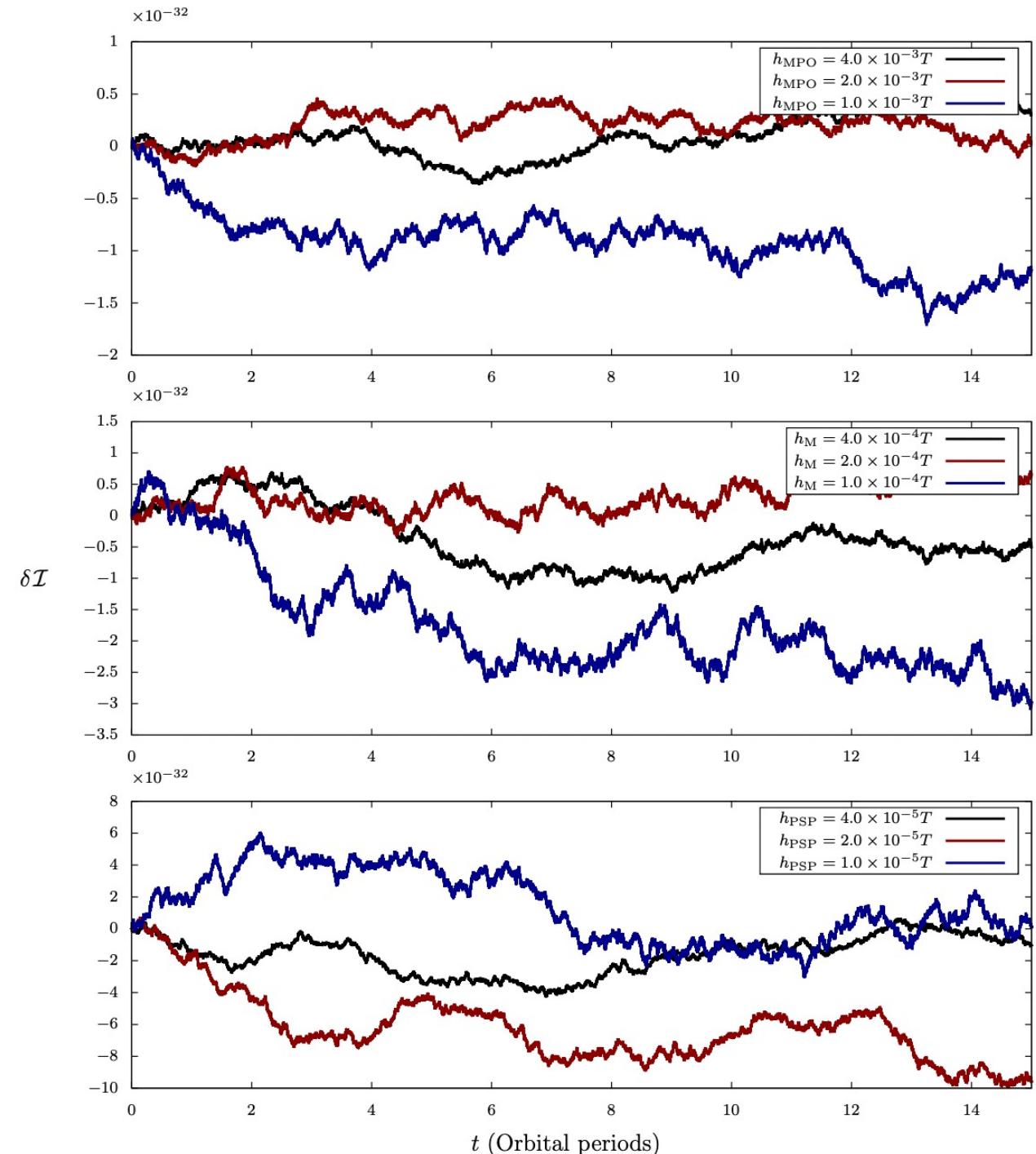
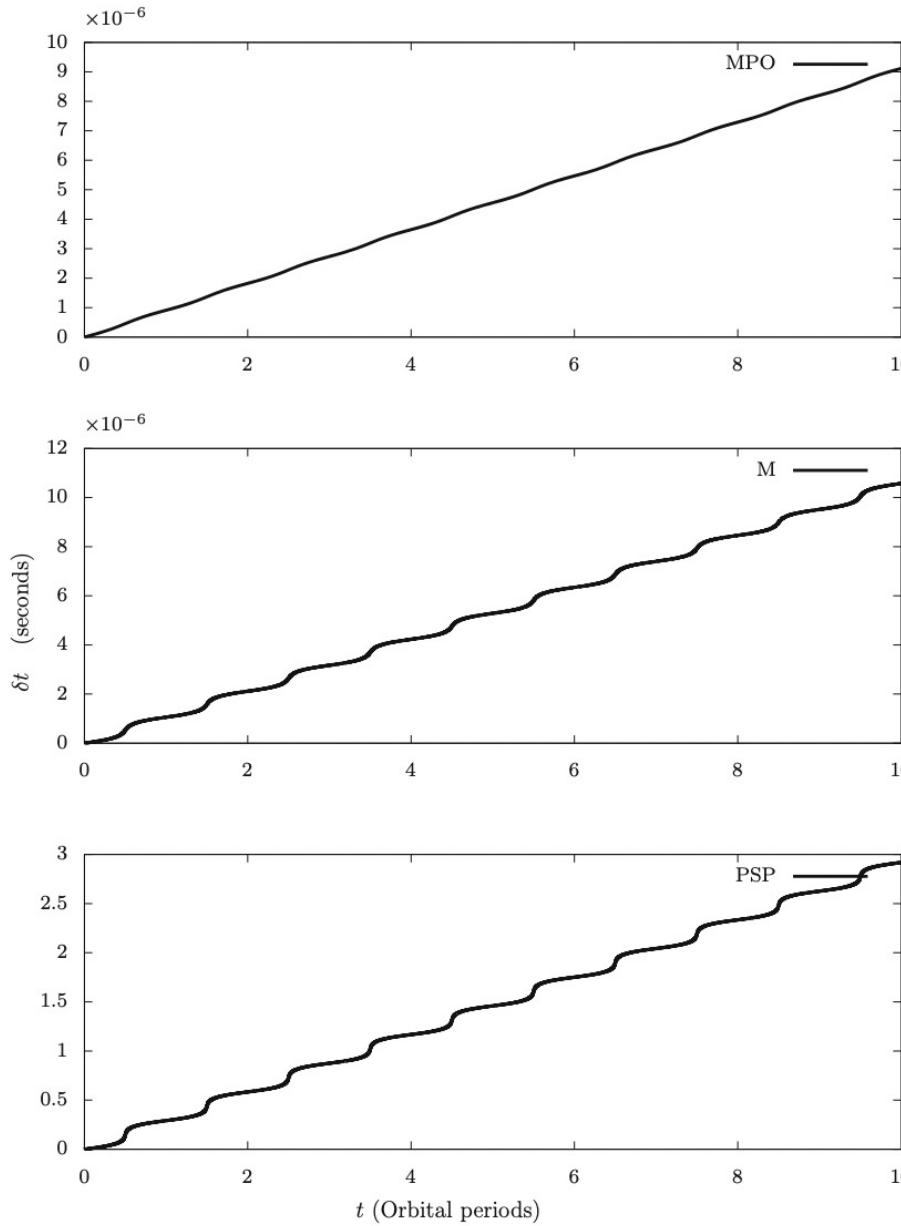
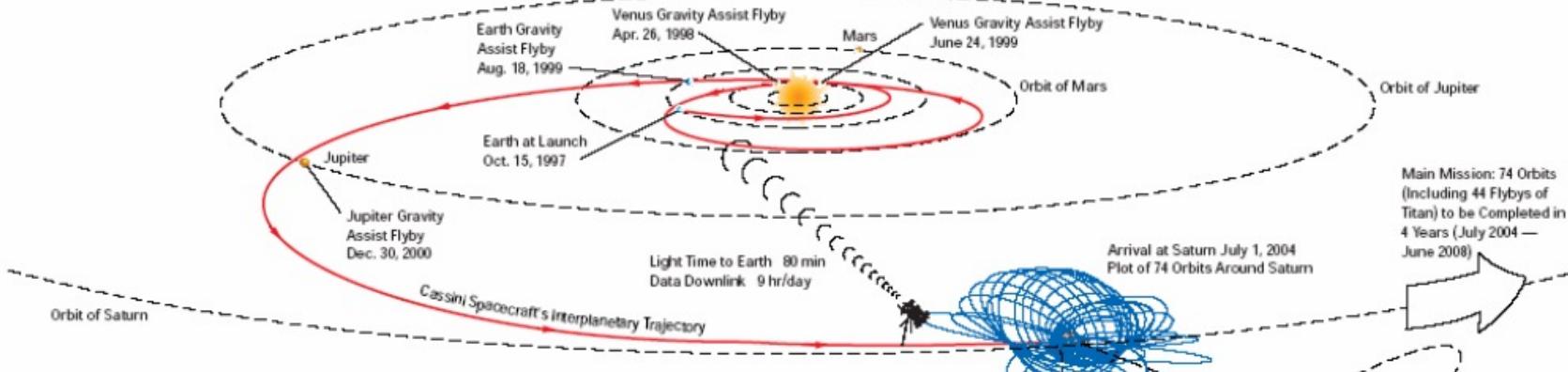
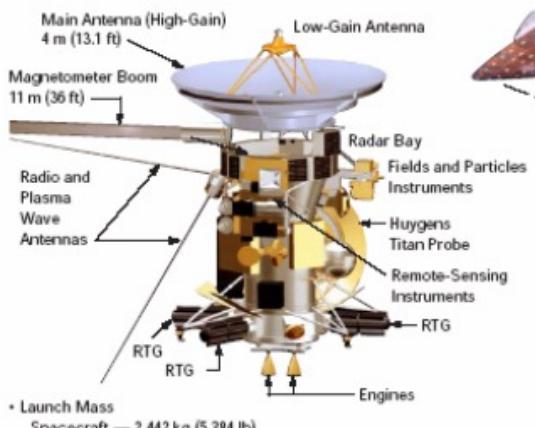


Fig. 1 Evolution of spacecraft proper time with global coordinate time where $\delta t \equiv t - \tau$

THE CASSINI MISSION TO SATURN

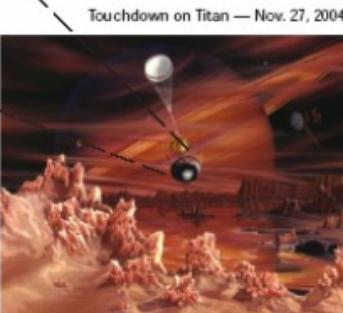


THE CASSINI SPACECRAFT



- Launch Mass:**
 - Spacecraft — 2,442 kg (5,384 lb)
 - Propellant — 3,132 kg (6,905 lb)
 - Total Mass — 5,574 kg (12,288 lb)
- Propulsion:** Two engines, 445 Newton (100 lb) thrust each
- Electrical Power Source:** Three radioisotope thermoelectric generators (RTGs)
- Optical Remote-Sensing Instruments:** Will determine temperatures, chemical composition, structure, and chemistry of Saturn, its rings, moons, and their atmospheres; will measure the mass and internal structure of Saturn and its moons; will photograph Saturn, its rings, and moons in visible, near-infrared, and ultraviolet wavelengths.
- Radar:** Will map Titan and measure heights of surface features.
- Field and Particles Instruments:** Will map the magnetic field of Saturn; detect charged particles and plasmas; study interactions between solid bodies and the solar wind; investigate ice and dust, plasma waves, and radio waves.

HUYGENS TITAN PROBE



During 3 hours of science observation and measurements, the Huygens Probe instruments will:

- Collect aerosols for chemical analysis.
- Make spectral measurements and take pictures of Titan's surface and atmosphere.
- Measure wind speeds using the Doppler effect.
- Identify constituents in atmosphere.
- Measure physical and electrical properties of the atmosphere.
- Measure physical properties of the solid or liquid surface of Titan.

CASSINI PARTNERS

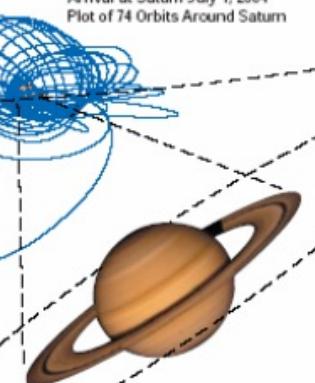
The Cassini mission is a joint effort of the National Aeronautics and Space Administration (NASA), European Space Agency (ESA), and Italian Space Agency (ASI). The mission is managed for NASA by the Jet Propulsion Laboratory, California Institute of Technology. Partners include the U.S. Air Force (USAF), Department of Energy (DOE), and academic and industrial participants from 19 countries.

Main Mission: 74 Orbits
(Including 44 Flybys of
Titan) to be Completed in
4 Years (July 2004 —
June 2008)

TITAN

SATURN'S LARGEST MOON

- Distance to Saturn: 1,221,850 km (759,200 mi)
- Diameter: 5,150 km (3,199 mi)
- Density: 1.82 g/cm³
(equivalent to 1.82 times the density of water)
- Surface Temperature: -181 °C
(-294 °F)
- Surface Pressure: 1.5 bars
(approximately 1.5 times surface pressure at sea level on Earth)
- Rings: 7
- Moons: 18
- Composition of Atmosphere:
Hydrogen (H₂)
Helium (He)
Methane (CH₄)
Ammonia (NH₃)
— and numerous other hydrocarbons and nitriles



SATURN

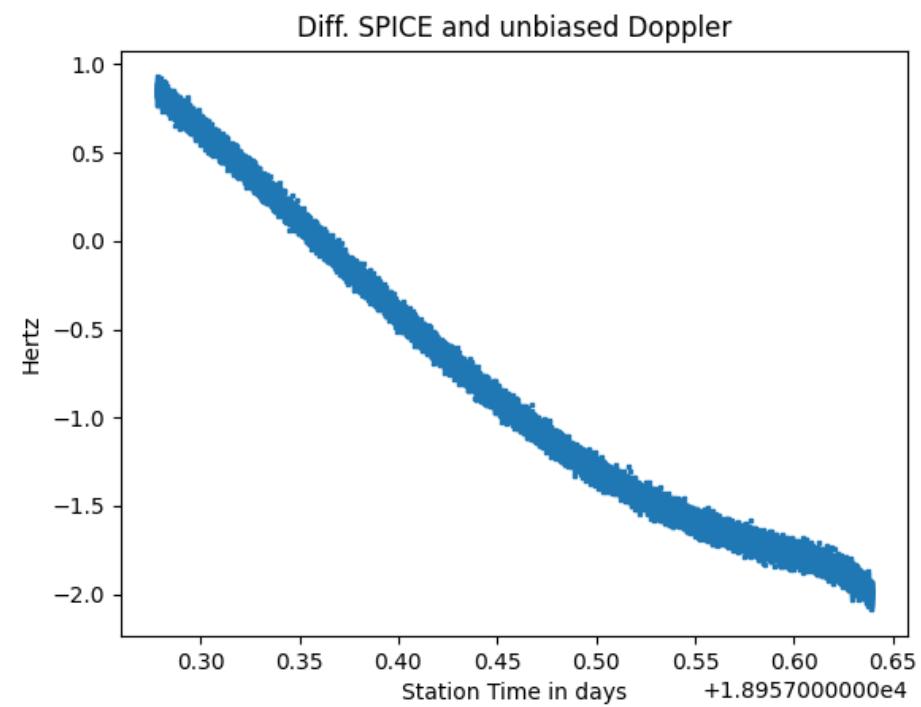
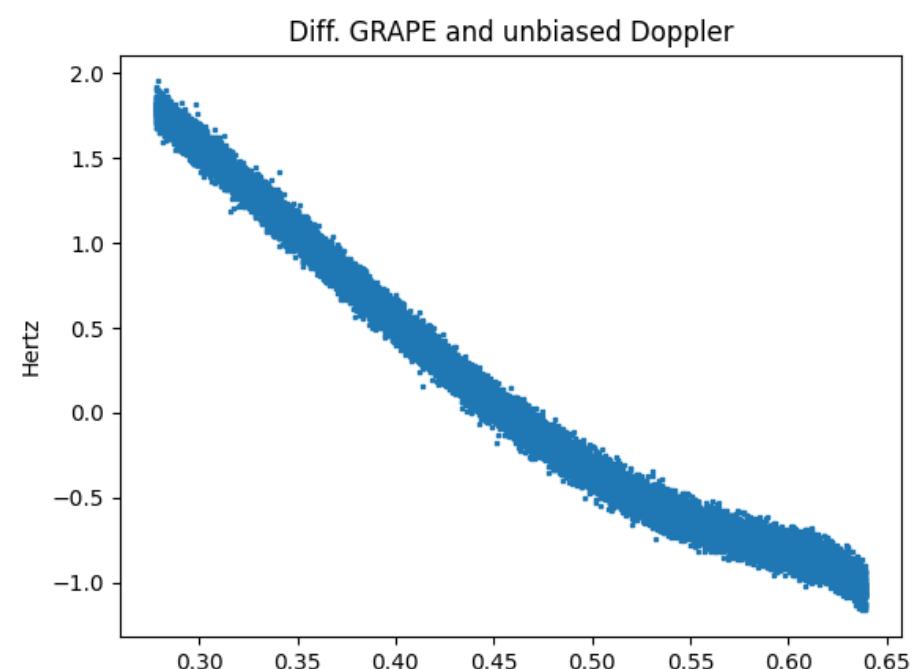
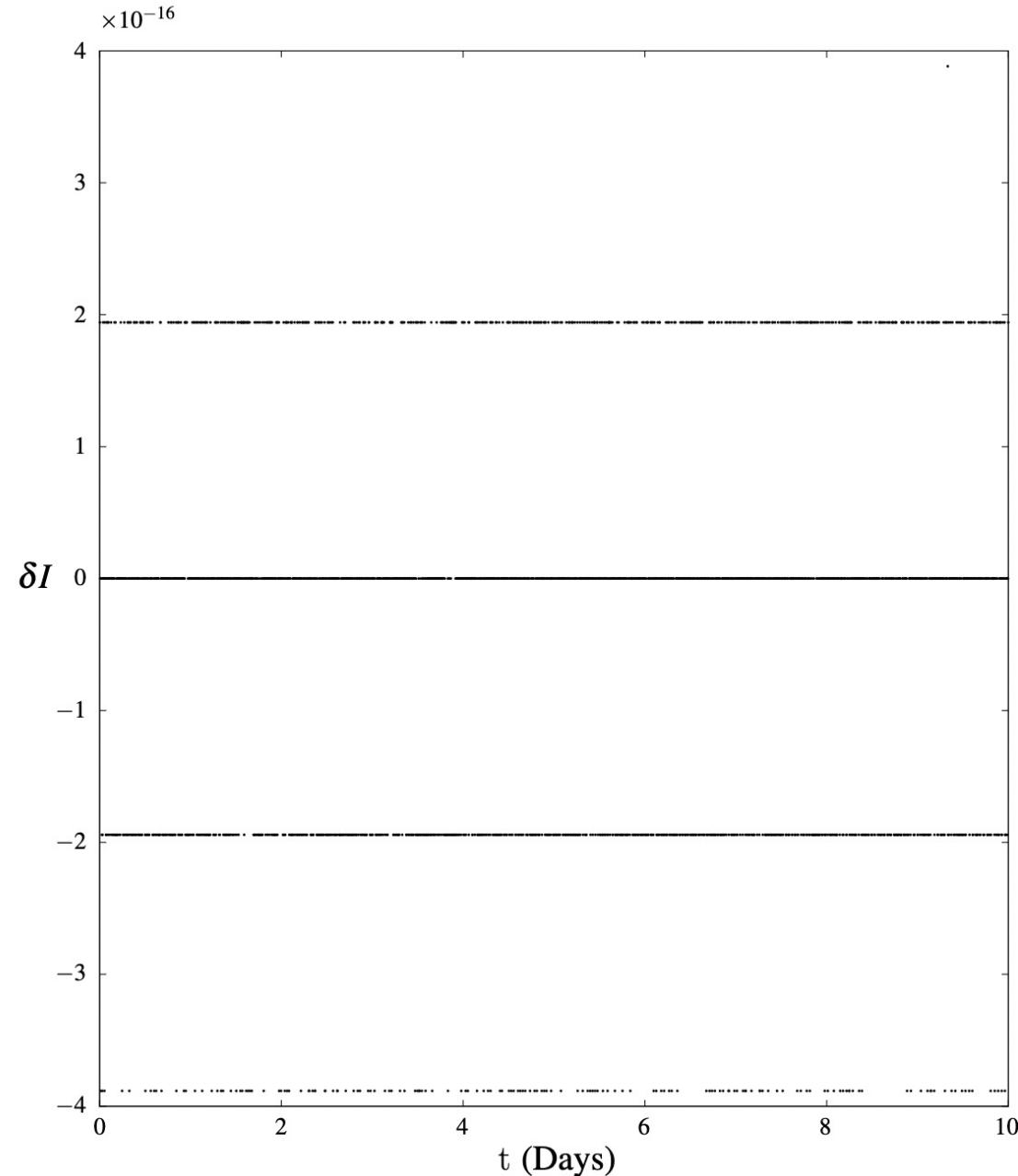
- Diameter: 120,660 km (74,975 mi)
- Density: 0.69 g/cm³
- Length of Day: 10 hr 40 min
- Length of Saturn Year: 29.42 Earth Years
- Rings: 18
- Moons: 18
- Composition of Atmosphere:
Hydrogen (H₂)
Helium (He)
Methane (CH₄)
Ammonia (NH₃)
— and numerous other hydrocarbons and nitriles

World Wide Web (WWW): <http://www.jpl.nasa.gov/cassini>



National Aeronautics and Space Administration
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California
JPL 400-843 10/99

GRAPE: A practical example (work in progress)



GRAPE: Moving forward

- Present practical extension of GRAPE with archival Cassini data at the ISSFD 2022
- Implement appropriate local force modelling using details given in Moyer 2000
- Correct SPICE library output for reference frame transformations and light time solutions using appropriate IAU recommendations and by numerically solving null geodesic equation (including effects due to plasma).
- Use GRAPE for PN parameter estimation (see Bertotti, 2003)
- Release GRAPE for public use (JOSS plus Github repo)

Backup slides GRAPE: A practical example explained (work in progress)

- Input metric: NASA's JPL n-body metric tensor components (See Moyer Sect 2, 2000)
- Set $f^\alpha = 0$
- Use GRAPE to generate ephemeris for duration of Cassini GWE
- Create GRAPE SPICE Kernel using NASA's mkspk executable
(https://naif.jpl.nasa.gov/pub/naif/toolkit_docs/FORTRAN/ug/mkspk.html)
- Use SPICE SPKEZR to obtain light-time solution
(https://naif.jpl.nasa.gov/pub/naif/toolkit_docs/FORTRAN/spicelib/spkezr.html)
- Correct SPICE light-time solution due to space-time curvature (See Moyer Sect 8, 2000)
- Generate pre-fit residuals for SPICE and GRAPE estimates.

Backup slides: Very brief biography

- PhD completed at the University of South Australia in 2019
- Research assistant with the Space Environment Research Centre (SERC)
 - Semi-analytic satellite theory
- Industry research fellow with EOS Space Systems
 - Nonlinear state and parameter estimation
 - Operational astrodynamics
- Recently started a postdoc at the University of Melbourne
 - Parameter estimation for HMXB systems in the Small Magellanic Cloud.



Backup slides: On the nature of non-gravitational forces in general relativity

$$T^{\alpha\beta} = \Theta^{\alpha\beta} + S^{\alpha\beta}, \quad (4)$$

where we define the stress tensor according to $S^{\alpha\beta}$ and we interpret $\Theta^{\alpha\beta}$ as the energy-momentum of an isolated test particle or system of non-interacting test particles defined in terms of mass density ρ and test particle four-velocity u^α according to $\Theta^{\alpha\beta} \equiv \rho u^\alpha u^\beta$.

It is well established that the conservation equation

$$\nabla_\alpha T^{\alpha\beta} = 0, \quad (5)$$

gives rise to the geodesic equation of motion in the *absence of stresses* ($S^{\alpha\beta} \equiv 0$). In the case where $S^{\alpha\beta} \neq 0$, we find

$$\nabla_\alpha \Theta^{\alpha\beta} = \rho \mathcal{K}^\beta, \quad (6)$$

where we have introduced the *force density* \mathcal{K}^β associated with an arbitrary non-gravitational force f^β and we have $\nabla_\alpha S^{\alpha\beta} = -\rho \mathcal{K}^\beta$. Eq. (6) is equivalently expressed as

$$\rho u^\alpha \nabla_\alpha u^\beta + u^\beta \nabla_\alpha (\rho u^\alpha) = \rho \mathcal{K}^\beta, \quad (7)$$

Thus, the complete system of equations of motion describing the perturbed motion of space-craft subject to an external perturbation f^α employed in GRAPE are given by

$$f^\alpha \equiv u^\beta \nabla_\beta u^\alpha = \mathcal{K}_\beta (g^{\alpha\beta} - u^\alpha u^\beta / c^2), \quad (9)$$